Name _____ Period_____

Equations

Keep the balance!

Whatever you do to the left, you have to do to the right!

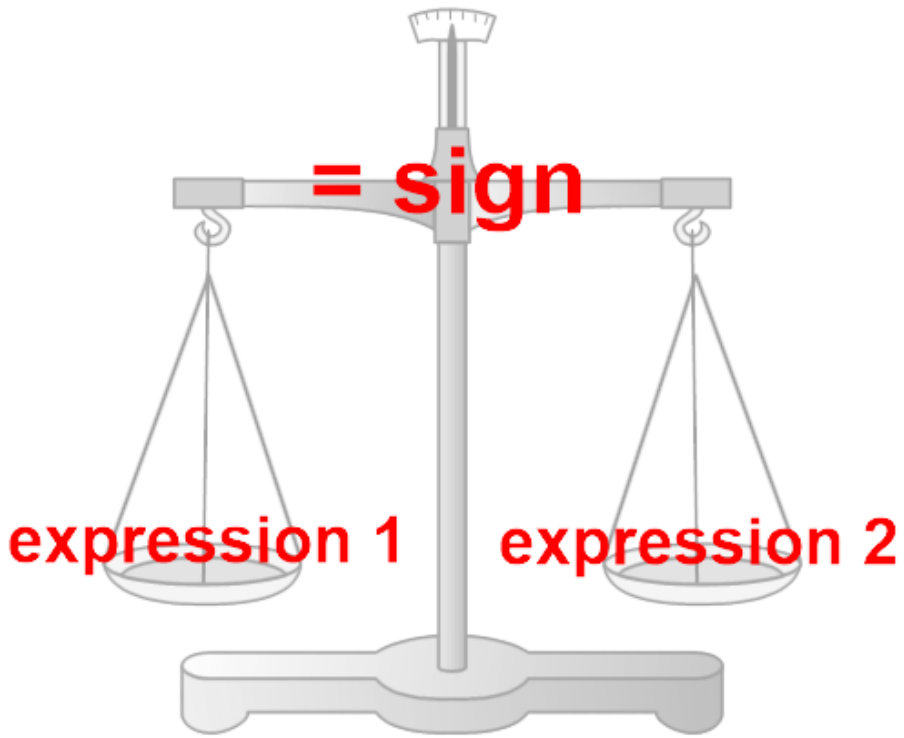
Name _____ Period _____

What is an equation?

An equation is a mathematical statement containing an equal sign to show that two expressions are equal.

An equation can be compared to a balanced scale.

Both sides need to contain the same quantity in order for it to be "balanced".



For example,

$9 + 11 + 4 = 6 + 7 + 11$
is an equation,
because both sides are equal.

$9 + 11 + 4 = 6 + 7 + 11$
$24 = 24$

Here are more examples:

$10 - 2 = 4 + 4$ $\quad$ $5(4) + 3 = 5^2 - 2$ $\quad$ $77 \div 11 = 9 - 2$

Check each example to see if it makes a balanced equation. If the equation is balanced, we'll put an equal sign between the expressions. If it is not balanced, we'll use this symbol: ≠ which just means not equal. Remember to use the order of operations (PEMDAS) still.

3.5 + 7 4.2 6^2 40 – 4 10 – 10 ÷ 2 $\frac{25}{5}$

4m + 6 2(2m +3) 14 – 5 3^3 63 ÷ 7 8 + 2

All of the key words you learned with expressions are still used in equations, but now we also need words to show where to put the equal sign.
Watch for these clues:

total the same value as comes to

is are was were will be

the same as sold for gives you

Let’s try writing equations from words:

four times a number is 12 _____

12 is 4 less than a number _____

a number divided by 12 gives you 4 _____

12 is the same value as a number plus 4 _____

The next step is to use these same skills to read word problems and write equations from them.

Alice has the 5 newest DVDs, which is 4 less than the amount Jon has.

Mike has $12, which is half as much as Paul has.

Jasmine, who bought $5 worth of candy, spent $3 more than Leah spent.

Tom is 14 years old, which is twice as old as his brother Chris.

Determining the Solutions of Equations

A solution to an equation is a number substituted for the variable that makes the equation true.

In order to determine if a number is a solution, replace the variable with the number and evaluate the equation.

If the number makes the equation true, it is a solution.
If the number makes the equation false, it is not a solution.

Which of the following is a solution to the equation:

$x + 17 = 21$ $\{2, 3, 4, 5\}$

Which of the following is a solution to the equation:

$m - 13 = 28$ $\{39, 40, 41, 42\}$

Which of the following is a solution to the equation:

$3p - 4 = 38$ $\{12, 13, 14, 15\}$

Solving Equations!!!

First we evaluated expressions, where we were given the value of the variable, and determined which solution made the equation true.

Now, we are told what the expression equals and we need to find the value of the variable.

When solving equations, the goal is to isolate the variable on one side of the equation in order to determine its value (the value that makes the equation true).

This will eliminate the guess & check of testing possible solutions.

In order to solve an equation containing a variable, you'll need to use inverse operations.

Inverse operations are operations that are ***opposites***, or undo one another.

Addition - _____

Subtraction - _____

Multiplication - _____

Division - _____

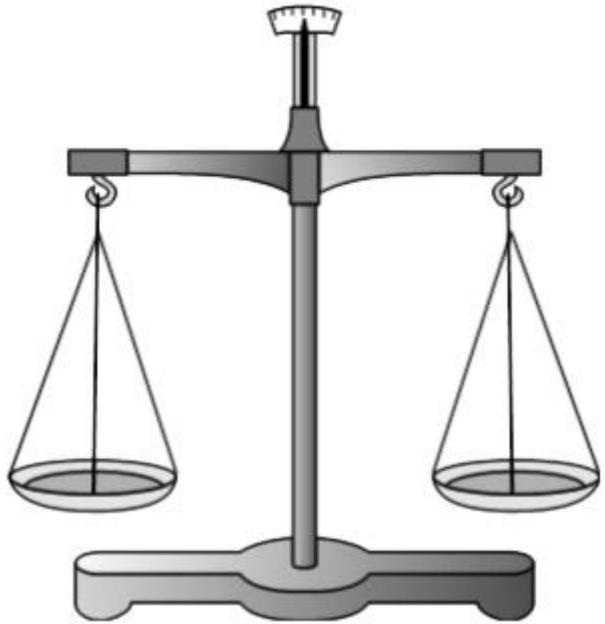
Write the inverse for each example:

+ 5 _____ x 2 _____

- 7_____ ÷ 4 _____

To solve equations, you must use inverse operations in order to isolate the variable on one side of the equation. Isolate means to get all alone.

Whatever you do to one side of an equation, you MUST do to the other side!



Keep the scales balanced at all times!!

Addition Property of Equality

It’s okay to add the same number to both sides of an equation!

$m - 6 = 10$

$v - 5 = 9$

$r - 30 = 50$

$h - 15 = 23$

$a - 2.5 = 6.0$

$p - 10 = 40$

Subtraction Property of Equality

It’s okay to subtract the same number from both sides of an equation!

$m + 6 = 11$

$12 + h = 25$

$g + 9 = 20$

$6 + j = 8$

$b + 2 = 6$

$f + 4.5 = 9.2$

Multiplication Property of Equality

It’s okay to multiply the same number to both sides of an equation!

$m \div 2 = 10$

$h \div 6 = 5$

$q \div 4 = 9$

$j \div 8 = 40$

$f \div 7 = 10$

$d \div 3 = 21$

Division Property of Equality

It’s okay to divide both sides of an equation by the same number!

$8y = 56$

$5r = 35$

$4g = 20$

$1.6w = 9.6$

$31.4k = 31.4$

$3m = 45$

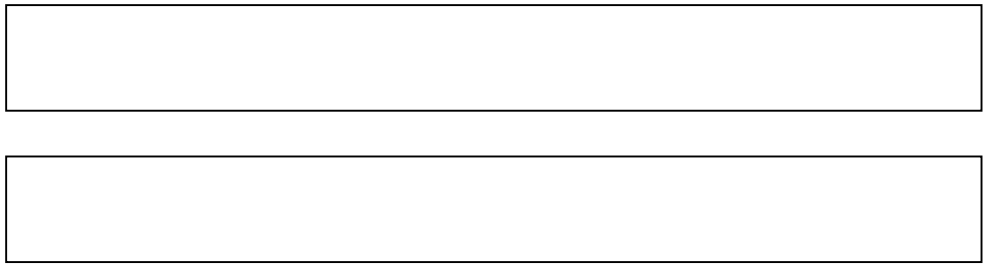
Using a Diagram to Represent an Equation

As we've seen in other topic such as dividing fractions, we have to learn how to solve the problem but we also need to learn how to show our thinking and work. This lesson will help you draw a diagram to show the solution to a one-step equation.

This all goes back to understanding that the equal sign means that the left side is exactly the same as the right side. Everything must stay balanced.

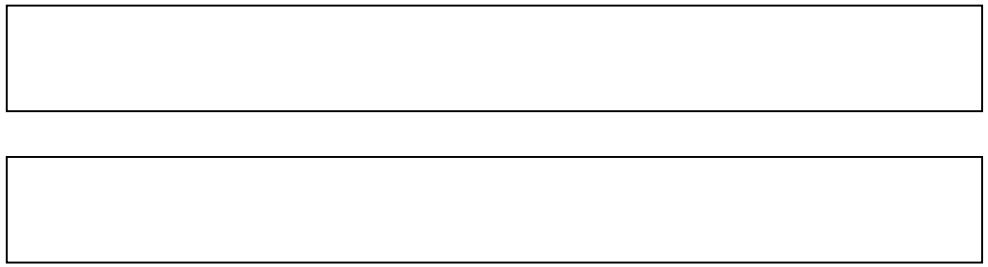
Tape Diagram Addition: $a + 2 = 8$

Draw two equal sized tapes.



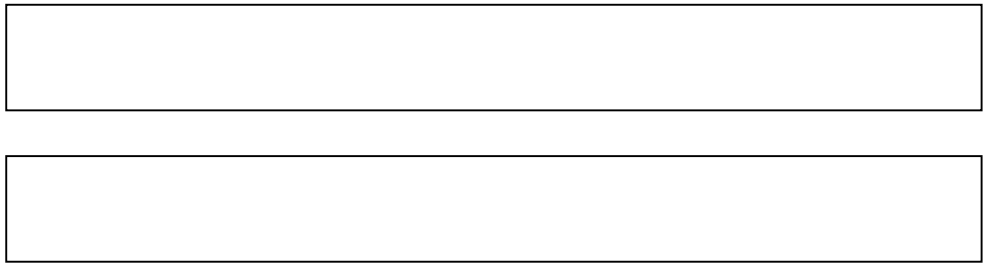
Tape Diagram addition:

$\boldsymbol{b} + 9 = 15$



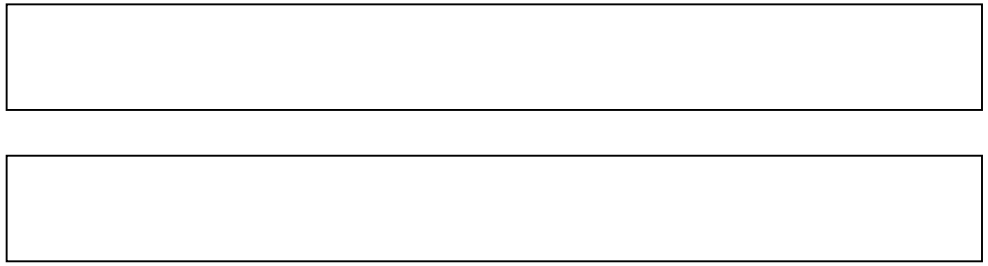
Tape Diagram addition:

$12 = 8 + \boldsymbol{c}$



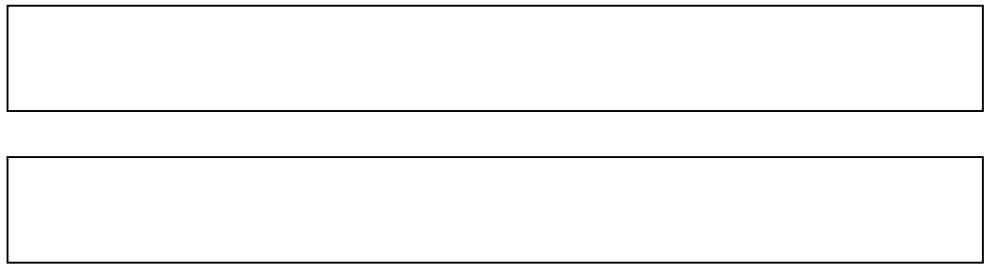
Tape Diagram subtraction: $m - 5 = 7$

In this example, it is easier to first solve for the variable, then draw the tape diagram to match the new equation.



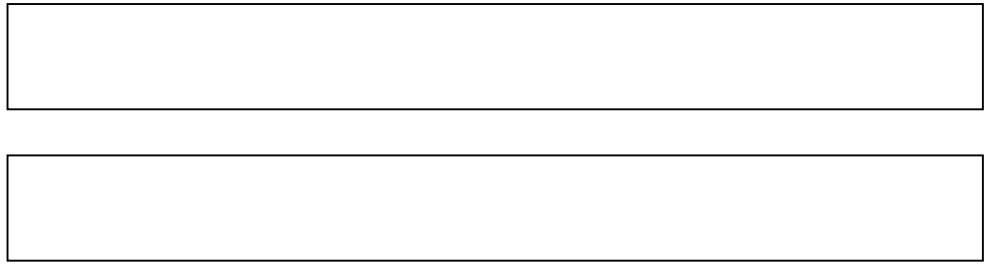
Tape Diagram Subtraction

v – 10 = 30



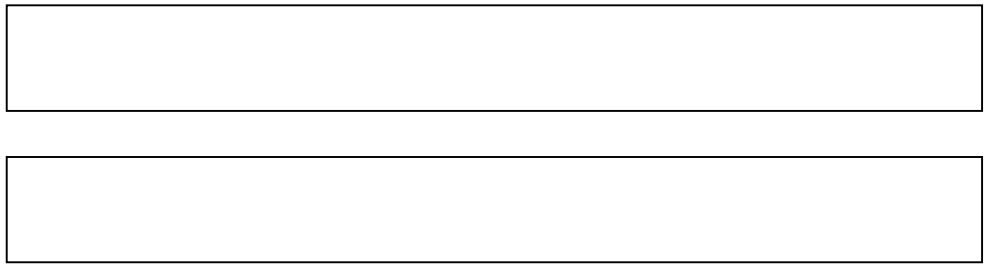
Tape Diagram Subtraction

b – 31 = 5



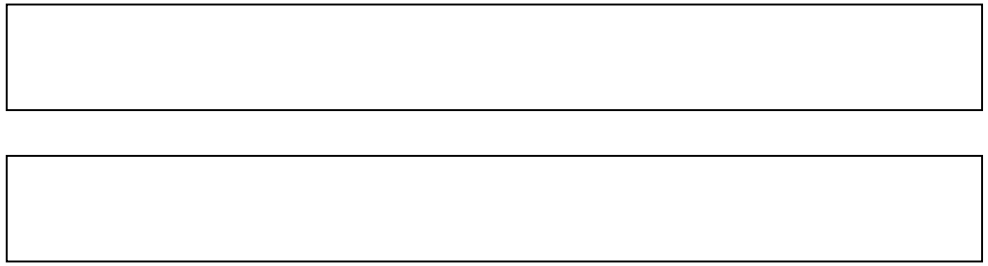
Tape Diagram: Multiplication

m(5) = 30



Tape Diagram: multiplication

g(4)=24

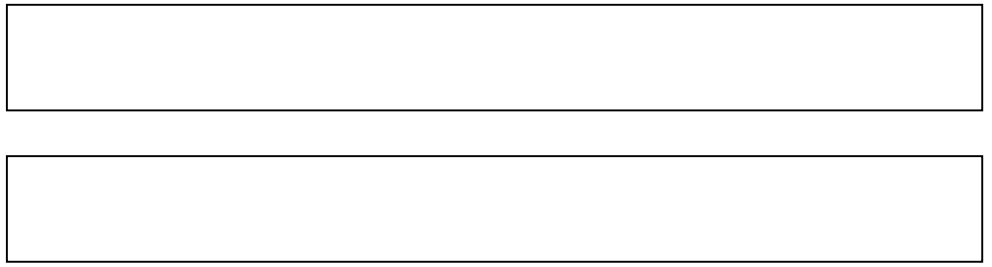


Tape Diagram Division

Again, it is easier to solve the equation first, then draw a model for the equation.

$m \div 7 = 8$

$y \div 4 = 12$



An **inequality** is a statement that two quantities are **not** equal.

A quick review of symbols used to show inequalities:

$m < 6$ a number less than 6. ex: -3,1,2,5

$m \leq 6$ a number less than or equal to 6. ex: -3,1,2,4,6

$m > 6$ a number greater than 6. ex: 7,9,10,45

$m \geq 6$ a number greater than or equal to 6 ex: 6,7,12,90

An inequality is a statement that two quantities are not equal. The quantities are compared by using one of the following signs:

Important Words	Sample Sentence	Equivalent	Translation
is more than	Trenton is more than 10 miles away.		d > 10
is greater than	A is greater than B.		A > B
must exceed	The speed must exceed 25 mph.	The speed is greater than 25 mph.	s > 25

Here are some more expressions you may encounter:

Important Words	Sample Sentence	Equivalent	Translation
cannot exceed	Time cannot exceed 60 minutes.	Time must be less than or equal to 60 minutes.	$t \leq 60$
is at most	At most, 7 students were late for class.	Seven or fewer students were late for class.	$n \leq 7$
is at least	Bob is at least 14 years old.	Bob's age is greater than or equal to 14.	$B \geq 14$

Try These:

1. 14 is greater than a
2. b is less than or equal to 8
3. 6 is less than the product of f and 20
4. The sum of t and 9 is greater than or equal to 36
5. 7 more than w is less than or equal to 10
6. 19 decreased by p is greater than or equal to 2
7. Fewer than 12 items
8. No more than 50 students
9. At least 275 people attended the play

Solution Sets

Remember: Equations have one solution.

Solutions to inequalities are NOT single numbers. Instead, inequalities will have more than one value for a solution.

$$m \geq -5$$

This would be read as, "The solution set is all numbers greater than or equal to negative 5."

Possible solutions would include:_____

Try these: Circle the solution(s) to each inequality.

$x > 11$ {9, 10, 11, 12}

$m < 15$ {13, 14, 15, 16}

$x > 34$ {32, 33, 34, 35}

Sometimes, we have to solve the inequality algebraically first, then choose the possible solutions.

$3x > 15$ {4, 5, 6, 7}

$6y < 42$ {6, 7, 8, 9}

Graphing Inequalities

Inequalities are graphed on a number line.

In order to graph an inequality, you need to do three things:

1. Draw a circle (open or closed) on the number from the inequality.
2. Extend the line in the proper direction.
3. Add an arrow to show that the line would continue on to infinity.

For example, the inequality m>5 would be graphed as:

Determining Whether to Use an Open or Closed Circle

- An open circle on a number shows that the number is not part of the solution. It serves as a boundary only.

  It is used with "greater than" and "less than".
  The word equal is not included.
  < >

- A closed circle on a number shows that the number is part of the solution.

  It is used with "greater than or equal to" and "less than or equal to".
  ≤ ≥

Determining Which Direction to Extend the Line

Extend Line to the Left:
If the variable is smaller than the number then you extend your line to the left (since smaller numbers are on the left).

Extend the line to the left in these situations:
> variable
variable < #

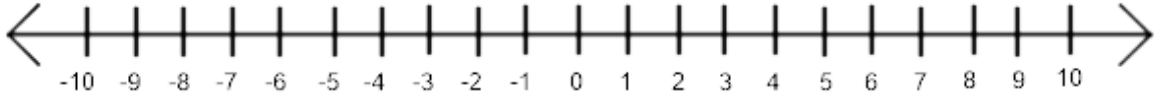
Extend Line to the Right:
If the variable is larger than the number then you extend your line to the right (since bigger numbers are on the right).

Extend the line to the right in these situations:
< variable
variable > #

Try these:

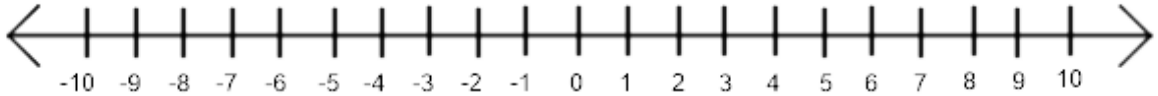
$m > 4$

$m < -2$



$m \geq -1$

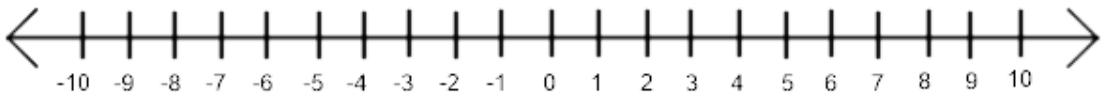
$m \leq 7$



$6 < m$

$-3 > m$

$1 \leq m$

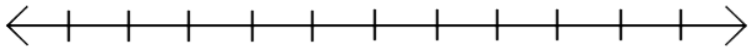


$-4 \geq m$

As always, we have to be able to apply what we have learned to real life situations. Let's have "story time" again, but this time we will be writing and graphing inequalities to go with each story.

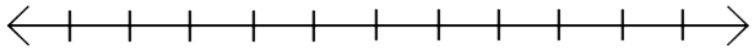
Tara is planning a birthday party. Her mom said she can invite to 10 people, plus herself.

_____



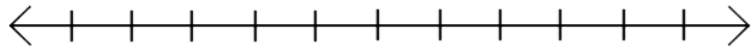
The baseball tournament will only be held if 20 or more people sign up.

_____



Today's temperature will not exceed 10 degrees below zero. Brrr!!

_____



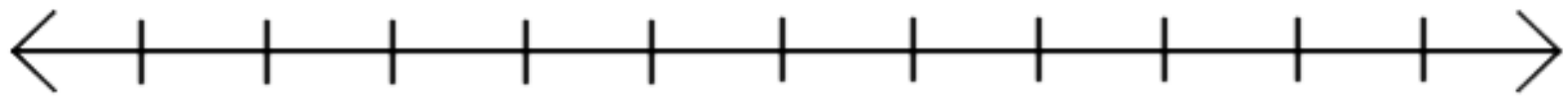
Let’s kick it up a notch! Sometimes inequalities can have limits on both sides of a variable.

For example, you could say, “Pick a number between 2 and 7.”

That could be shown as $2 \leq m \geq 7$

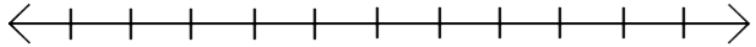
Possible solutions would be 2,3,4,5,6,7

This is what it would look like graphed on a number line:

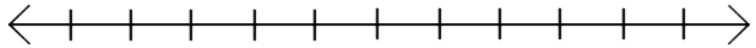


Let’s try a few.

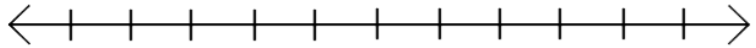
Amanda has between 5 and 10 goldfish. _____



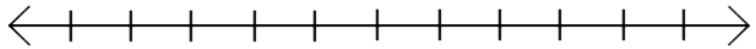
Carol is between 35 and 45 years old. _____



The temperature will be between ten below zero and five above zero. _____



The speed limit is between 40 mph and 65 mph. _____



Dependent and Independent Variables

Independent Variable- The variable that can represent different values. When we use variable to create graphs, this will be the “x” value. It will be the first column in a table.

Dependent Variable- The variable that’s value is the result of evaluating an expression involving the independent variable. This will be the “y” value. It will be the second column in a table.

For example, at the petting zoo, the zookeeper needs to know how much to feed the sheep. Each sheep needs 4 pounds of food each day. The independent variable is the number of sheep. The zookeeper needs to know the number of sheep first so she can determine how much food is needed.

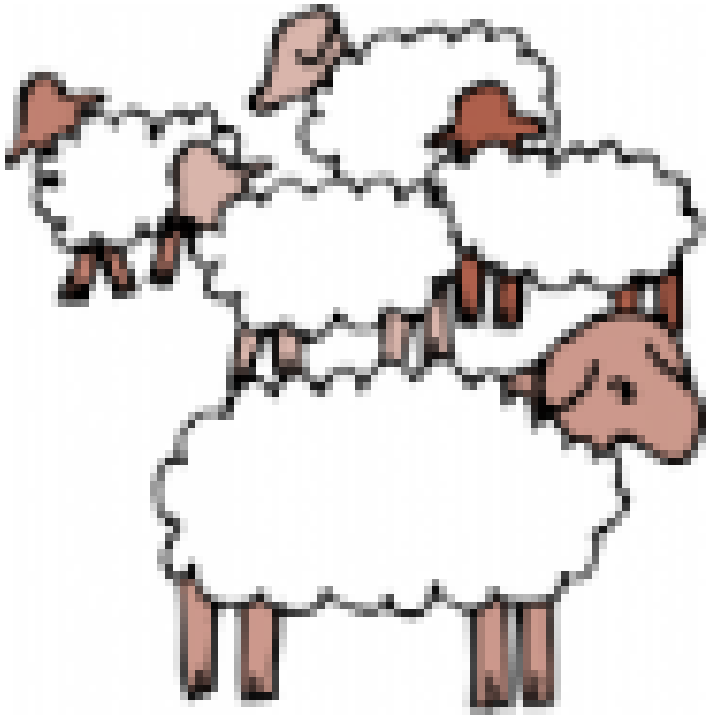
X is the number of sheep. It is the independent variable.

Y is the number of pounds of food needed for all of the sheep. It is the dependent variable.

The equation to represent this story would be:

$$y = 4x$$

The pounds of food = 4 times the number of sheep



This is what the equation would look like in a table.

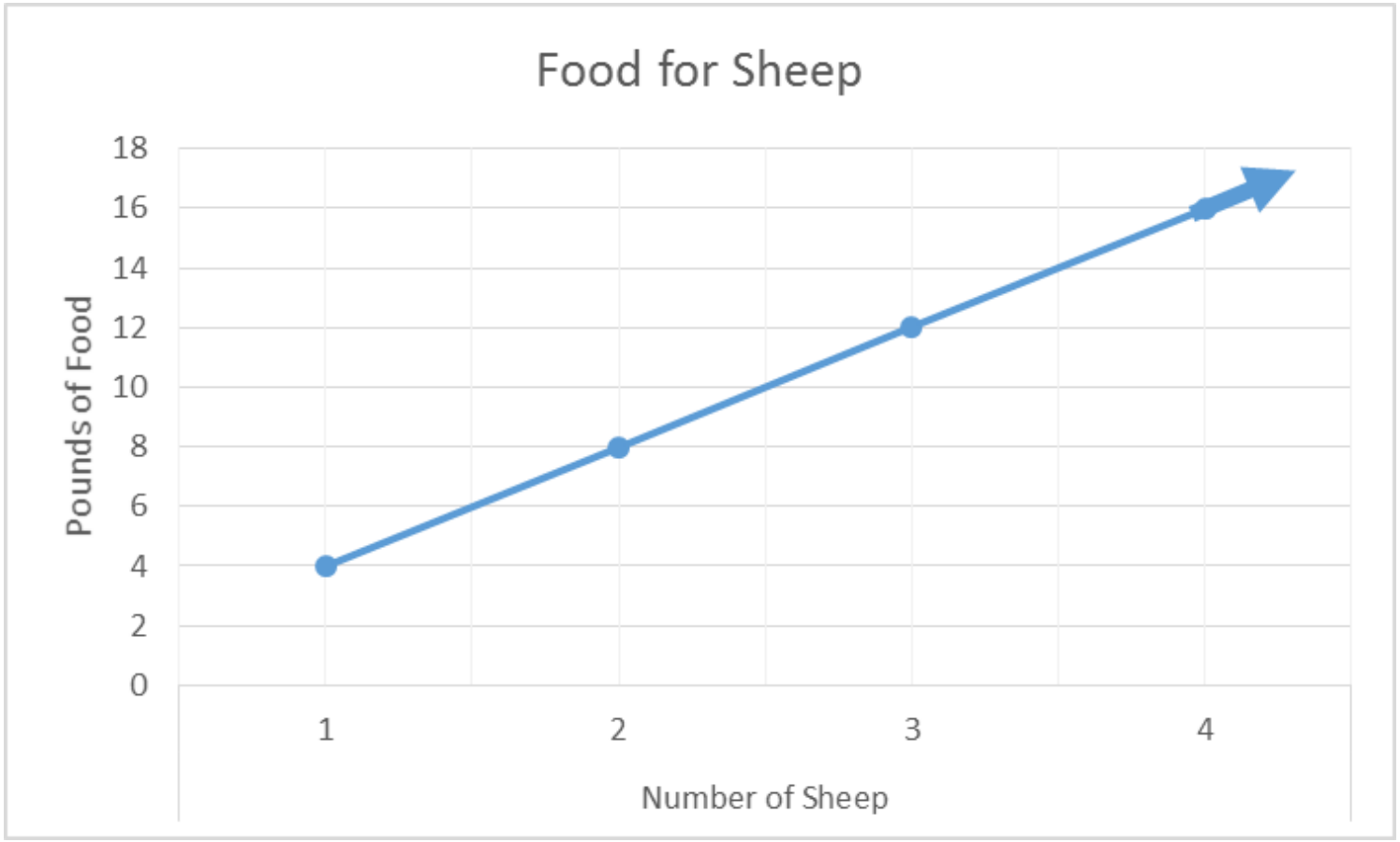
x **Number of sheep**	**y** **Pounds of food needed**
1	**4**
2	**8**
3	**12**
4	**16**

The next step is to take the numbers from the table and turn them into ordered pairs for graphing.
These would be the ordered pairs:

(1,4)
(2,8)
(3,12)
(4,16)

The last step is to turn the ordered pairs into a line on a graph.
Here are the parts you will need:

- Title
- X-axis title
- X-axis numbers
- Y-axis title
- Y-axis numbers
- Ordered pairs connected with a straight line and at least one arrow.



Let's try another story. Mrs. Hasbrouck's science classes planted bean seeds. The seeds germinated and the plants grew quickly. Every week, the plants grew another 12 cm taller. Show how tall the plants will be in 5 weeks using an equation, a table and a graph.

Independent variable _____ dependent variable_____

Equation_____

Table:

x	y

Ordered pairs:

_____

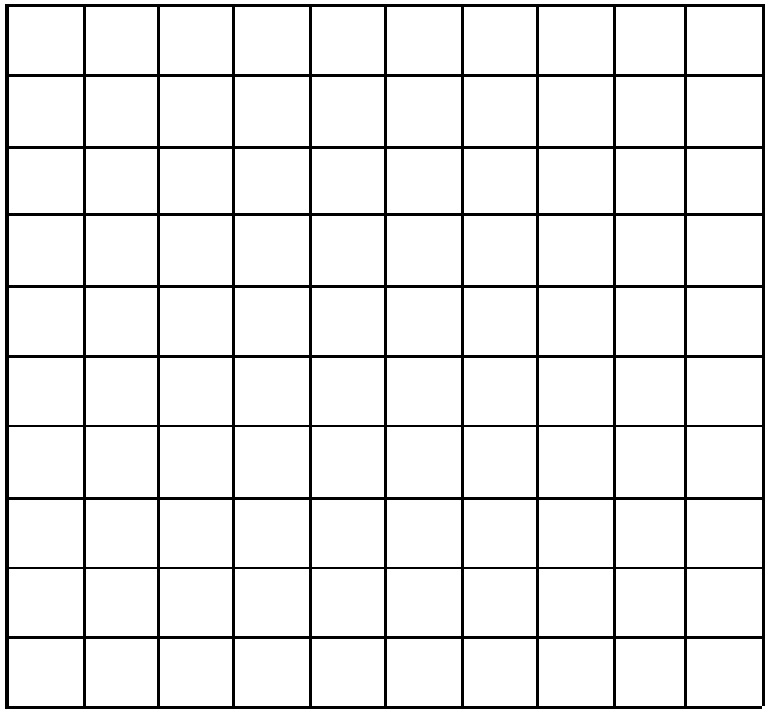
_____

_____

_____

_____

Graph:



Stephanie's graduation picnic costs $7 for every person. Show this relationship using an equation, a table and a graph.

Independent variable _____ dependent variable_____

Equation_____

Table:

x	y

Ordered pairs:

_____

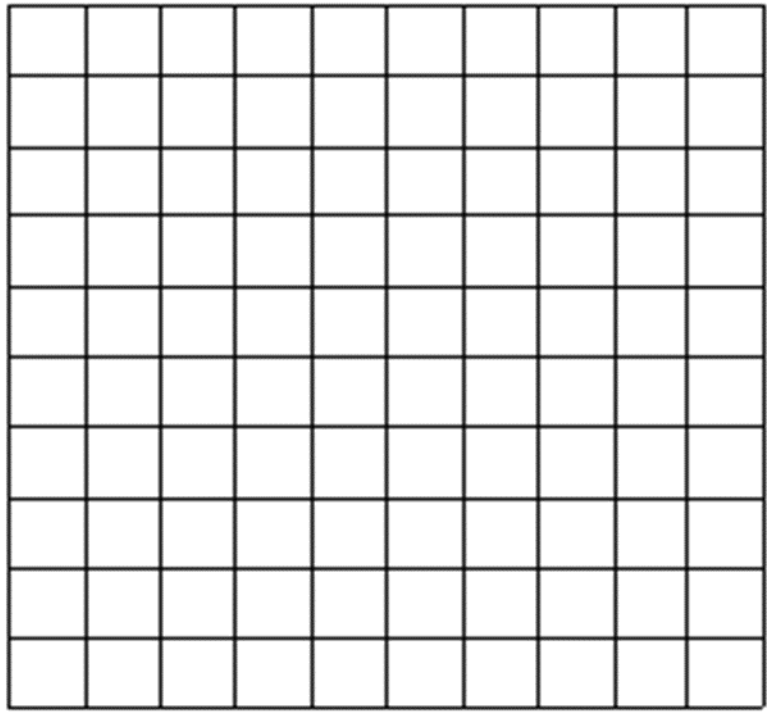
_____

_____

_____

_____

Graph:



Forensic scientists can use the length of the thighbone or femur in centimeters to estimate the height of a skeleton. One equation they may use is h = 2.6f + 65, where h represents the height and f represents the length of the femur in centimeters. Show this relationship using an equation, a table and a graph.

Independent variable _____ dependent variable_____

Equation_____

Table:

x	y

Ordered pairs:

_____

_____

_____

_____

_____

Graph: